Effects of Melting Upon Thermal Models of the Earth

Ray T. Reynolds, Peter E. Fricker, and Audrey L. Summers

Space Sciences Division
Ames Research Center, NASA
Moffett Field, Calif.

[redacted]

ABSTRACT

Theoretical and experimental evidence is presented for the existence of melting within the outer layers of the Earth. Numerical techniques are developed that permit the calculation of temperature distributions within a spherical body which melts according to a given melting curve. The possibility of fluid convection is discussed and the numerical procedures are modified to include the limiting case of highly efficient fluid convection. Models are constructed employing these techniques, and further application of these methods to the problem of the distribution of radioactive isotopes within the Earth is indicated.

INTRODUCTION

The method of constructing mathematical models for studying the thermal history of planetary bodies has been developed considerably within recent years. The purpose of this work is to extend the range of available theoretical models to include models which consider the effects of melting and fluid convection within a planetary body, with particular reference to the outer layers of the Earth.

Some early ideas of thermal history (Kelvin, Jeffreys) visualized an Earth condensing at high temperatures to a liquid, and then cooling rapidly to the solid state. The cooling was assumed to take place by fluid convection, with radiation from the surface and solidification taking place from the interior outward. Current theories of planetary

[redacted]

formation, as developed in the work of Spitzer, Hoyle, Urey, Schmidt, and others, involve the accumulation of planetary bodies as solids at relatively low temperatures. Allen and Jacobs [1956], Lubimova [1958] MacDonald [1959], and other authors have constructed models of the Earth, Moon, and other planetary and asteroidal objects, based upon the equation of heat conduction within a solid body having internal heat sources. They have calculated a wide variety of thermal models and have investigated the consequences resulting from variation of the parameters and assumptions required in calculating these models. The general conclusion reached was that temperatures within the Earth increase with time for depths below a few hundred kilometers. Since the effects of melting are to be considered here, it must first be determined that these rising temperatures do exceed the melting temperature at some time within the Earth.

TEMPERATURE DISTRIBUTION

Any attempt at a study of the Earth's thermal history encounters the very intractable problem of the initial temperature distribution. This problem is traditionally handled by listing the possible energy sources, discussing them briefly and semiquantitatively, and then 'estimating' a temperature distribution. This unsatisfactory state of affairs is only slightly ameliorated by the fact that for this study only a lower bound need be determined to indicate the existence of melting. This so-called 'initial temperature' is generally considered to be derived from three major sources; the accretional energy retained during planetary formation, the compressional energy released within the interior, and the possible contribution of short-lived radioactive isotopes. This is assuming a relatively short period of formation in which long-lived radioactives would not contribute significantly. Other sources have been suggested such as tidal friction [Kaula, 1964] and free radicals [Urey and Donn, 1956] but are rather unlikely to be significant energy sources. The initial temperature sources provide additional heating above the equilibrium temperature provided by solar radiation.

The total amount of accretional energy may be estimated by computing the gravitational potential energy of the Earth. With the Solution I density distribution given by Birch [1964], this potential energy may be calculated as 4.20×10^4 joules/g. This energy is much more than would be required to melt the entire Earth and obviously only a small fraction of the energy could be retained. The amount of energy retained depends primarily upon the rate of accretion.

Benfield [1950] considered the problem of accretional and compressional energy and discussed earlier work. Safronev [1958] and Lubimova [1955] estimated the temperatures within the Earth after the period of formation was completed. Lubimova [1958] estimated the temperature increase at 1270 km depth as between 200°C and 1200°C. MacDonald [1959] considered a constant temperature of 1300°C as appropriate for an accreting Earth in which equilibrium is maintained between the input of gravitational energy and the output of radiation.

The contribution due to extinct short-lived radioactives is dependent upon the amount of time between formation of the radioactive materials and the formation of solid bodies large enough to retain heat. Fish, Goles, and Anders [1960] have indicated the possible importance of this source for the meteorite parent bodies and contributions of as high as 2000 - 3000°/g are postulated. This energy source is extremely sensitive to the rate of accretion and hence no meaningful estimate of its importance can be made for the Earth.

Upon completion of the interval of planetary formation and the rapid decay of any short-lived radioactive materials, further temperature increases must come primarily from decay of long-lived radioactives, principally isotopes of K, Th, and U, and from energy released during rearrangements of the internal density distribution. For an Earth initially uniform in composition and composed primarily of iron and silicate phases, the rising temperatures will exceed the melting curve of iron [Strong, 1962], within the deep interior, before the silicate phase begins to melt. Formation of an iron core by some irreversible process such as that outlined by Elsasser [1963] will result in the release of large quantities of energy. Urey [1952] has estimated this energy source to be as high as 2000°/g. Since the bulk of this energy is released before the melting temperatures of the silicate phase are reached, the contribution of energy from core formation is considered to be contained within the initial temperature estimate for the purposes of these calculations. Although a good deal of evidence can be presented for the existence of a reasonable lower bound to the initial temperature, too much uncertainty remains to permit the presentation of this lower bound as more than an estimate. MacDonald [1963] gives a value of almost 1800°C as a low initial temperature for 1500 km depth and Birch [1965] assumes that the initial temperature distribution is close to the melting curve of iron. We estimate 1500°C to be the lower limit to the initial temperature at depths greater than a few hundred kilometers.

The temperatures within the Earth also depend upon the rate of removal of heat energy. The models considered here include the effects of heat transfer by radiation and lattice conductivity.

The possibility of heat transfer by convection within material at temperatures below its melting point has been discussed recently [Vening-Meinesz, 1964]. Knopoff [1964], however, has shown that mantle-wide solid convection will be inhibited by the inhomogeneities within the transition zone. Deeper within the mantle the temperatures should fall far short of the melting point. The possibility of some solid-state convective heat transfer by means of local cells cannot be completely ruled out. However, if these cells do exist they should be limited to a relatively narrow region between the transition zone and the coolest outer-most layers, and the efficiency of this process in transporting heat is questionable [MacDonald, 1963]. Hence, solid-state convection is not considered in the calculations.

For given values of the initial temperature distribution, radioactive abundances and distributions, thermal conductivities, and other necessary parameters, the temperature distribution may be calculated as a function of time for a model planet, in the solid state, by means of the heat conduction equation.

MELTING TEMPERATURES

An estimate of the melting temperatures as a function of pressure within the Earth's mantle is now required. It must be emphasized, of course, that such a complex multicomponent system as the Earth's mantle will not melt at a constant temperature for a given pressure. Therefore, the range of temperatures over which melting occurs should be obtained, rather than the simple melting curve which would be expected for a single component crystalline solid. Minor fractions can be expected to melt over an extremely wide range, e.g., H_2O and graphite, but fortunately the temperature interval between solidus and liquidus is only of the order of a few hundred degrees for the major fraction of many multicomponent silicate systems. Alexeyeva [1958] reports the melting of six stony meteorites, mainly chondrites, as occurring between 1180 - 1350°C. In fact, the major fraction of mantle material may well be melted or crystallized over an even narrower range. From a study of the Stillwater Complex, e.g., Hess [1960] reports that 60% of a basaltic melt crystallized over a temperature interval of only 25°C. For a system of this type, the assumption of a single component with a constant melting temperature provides a good approximation.

Experimental melting point data are available for selected silicate materials only to pressures of some 50 kb and extrapolation of these data, particularly through a region as inhomogeneous as the transition layer, is an extremely unreliable process. Uffen [1952] presented a curve (Figure 1) derived from seismic data and based upon treatment as a single component system using Lindemann's melting criterion and the expression for the Debye characteristic frequency. More recently Clark [1963] has criticized Uffen's curve and has constructed a melting curve (Figure 1) to account, in an approximate manner, for the fact that the mantle is a multicomponent system. Clark's curve describes the minimum melting temperature for the multicomponent system. The dashed curve indicates the maximum melting curve for Clark's mantle on the assumption that the range of melting is a constant 180°.

Fig. 1

The low pressure data for the melting range of basalt-eclogite from Yoder and Tilley [1962] are in general agreement with the melting temperatures indicated by the Clark and Uffen curves, while melting curves for single component materials such as diopside [Boyd and England, 1963] enstatite [Boyd, England, and Davis, 1964] and forsterite [Davis and England, 1964] are considerably higher.

The approximate character of these curves is duly noted, but while they may not be accurate in detail they do indicate the region in which melting should occur and thus provide a reasonable basis for calculations.

CALCULATION OF MELTING MODELS

A number of thermal models were calculated, to investigate numerically the possibilities of melting, using these melting curves. The basic equation of heat conduction for a spherically symmetric body with internal heat sources,

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 K \frac{\partial T}{\partial t} \right\} + A$$

where r radius

t time

and the following are functions of radius and time

T temperature

K thermal conductivity

A rate of heat production per unit volume

C_p heat capacity

was employed together with a relation for the thermal conductivity given by MacDonald [1964] as,

$$K = c + \frac{16 n^2 s T^3}{3\epsilon}$$

where c lattice conductivity

n index of refraction

s Stefan-Boltzmann constant

ϵ opacity

The internal heat sources are due to the decay of radioactive isotopes and are expressed by the relation

$$A = \sum_{j=1}^{l} B_j w_j \, e^{-\lambda_j t_p}$$

where j specific radioactive isotope

l number of radioactive isotopes

B radioactive heat generation rate per g of isotope

w isotope weight fraction

λ decay constant of isotope

The effect of the electrical conductivity on the opacity is generally small for the temperatures considered here and may be neglected. The numerical techniques employed follow procedures similar to those outlined by MacDonald [1959] with the exception of a modification to include the latent heat of fusion. Levin [1962], in an interesting paper on the thermal history of the Moon, has predicted the occurrence of melting by means of a scheme which involves increasing the heat capacity within a melting region by a quantity sufficient to include the latent heat of fusion. He has assumed the melting to occur linearly with temperature over the melting region. We adopt a different assumption, namely, that the melting occurs at a constant temperature for a given pressure. The quantity of latent heat absorbed as a function of temperature across the melting region is not generally known. However, for systems such as the basaltic magma described by Hess [1960], the assumption of constant melting temperature describes the data more accurately than the linear melting assumption. In addition, the constant melting assumption lends a useful degree of stability to the numerical solutions.

A program was written to provide for the normal numerical solution of the heat conduction equation, except for intervals in which the melting temperature was reached or exceeded. When the melting curve temperature was exceeded for a given radius interval, the temperature was reset to the melting temperature by the program and the excess heat energy subtracted from the heat of fusion for that interval. When the heat of fusion had been reduced to zero, the temperature of the completely molten region was again allowed to rise. The same process occurred in reverse if a molten region was cooled below the melting point. This procedure is outlined in the appendix.

Since the actual abundances of radioactive materials within the Earth are uncertain, two sets of models with uniformly distributed radioactives were constructed, one using the chondritic abundances as given by MacDonald [1959] and the other using the so-called terrestrial abundances given by Wasserburg et al. [1964]. These abundances are listed in Table 1 and consequences of the assumption of these abundances were discussed by MacDonald [1964] for non-melting models. Birch's [1964] Solution I was used for the density distribution and was assumed to remain constant in time. Table 1

The estimated lower bound for the initial temperature distribution employed for these models rises parabolically from 0°C at the surface to 1500°C at 500 km depth and remains constant at 1500° from 500 km inward. This initial temperature includes core formation energy and any energy released by the decay of long-lived radioactives prior to the beginning of the calculations. The age of the Earth, after formation, is taken as 4.5 billion years for these models.

The amount of radioactive heat produced in the outer layers of the Earth is considerably underestimated by the assumption of uniform distribution of radioactives. The core is generally considered to be depleted in radioactives and movement of all radioactive materials from the core into the mantle will increase the radioactive heat output within the mantle by almost 50%.

Table 2 lists some of the parameters chosen for the construction of several representative models. Table 2
<u>MacDonald</u> [1959] has presented an extensive discussion of these parameters and the values used here, except for the heat of fusion, are chosen after his model 14. An average value of 400 joules/g was used for the heat of fusion based upon the data compiled by <u>Birch, Schairer, and Spicer</u> [1942].

Figure 2 shows the extent of melting under these assumptions for Fig. 2
the outer 1500 km of uniform chondritic Earth for both Clark's minimum melting curve and Uffen's curve. The standard nonmelting model is included for comparison. For the model with Clark's melting curve, the thickness of the molten layer was 740 km after 4.5 billion years, while the corresponding value with Uffen's curve was 320 km.

Results of calculations using the terrestrial abundances are given in Figure 3. With Clark's melting curve the molten zone amounted to 500 km but the melting point was not reached for a model with the Uffen curve where the temperatures failed to reach the melting curve by only 100°. Since the initial temperatures and heat production rates were deliberately held to low values, it is concluded that for the Clark and Uffen type curves, melting will take place within an Earth initially uniform with regard to the distribution of radioactive heat sources. Fig. 3

FLUID CONVECTION

The existence of large molten regions within the transition zone and upper mantle will require the consideration of heat transfer by convection since fluid convection is not inhibited by the inhomogeneities within the transition zone. Jeffrey's [1962] estimate that the melting point gradient exceeds the adiabatic gradient by approximately 1°/km was found to hold for the melting curves considered here and hence convection will occur within the molten zone.

The theory of convection is not sufficiently well understood to permit explicit calculation of the heat-transfer process. However, heat flux in the interior is low and the efficiency of fluid convection as a mechanism for heat transport is extremely high under most conditions. Hence the first approximation is to assume the limiting case of efficient convection, i.e., that convection carries through the molten region all heat above that amount required to slightly exceed the melting point. The melting temperature curve will be followed closely since, if the temperature drops below the melting point, solidification begins and convection slows down, raising the temperature again. If the temperature increases above the melting curve, convection increases and the temperature is lowered.

This process of simulated fluid convection was carried out by modification of the previously described program. These modifications fix the upper bound of temperature, within a given radius interval, at the melting point and move all excess energy into the adjacent outer radius interval while maintaining conservation of energy. This process is described in the appendix. Models embodying melting and this simulated convection were calculated using the modified Clark melting curve (Figure 1) for both chondritic (Figure 4) and terrestrial (Figure 5) models. Fig. 4,5

The regular nonmelting models are compared with models including melting only and models employing both melting and simulated convection. The temperature distributions for these models are given in Table 3. Table 3 The convecting models have a slightly smaller molten zone and one which is displaced toward the surface. A steeper thermal gradient in the outermost layers results in a larger loss of heat from the surface than for the nonconvecting models and the temperature distributions are significantly different.

For these models, a radius interval of 20 km was employed with corresponding time intervals of about 8 million years. The numerical solutions are stable to the usual checks for accuracy, such as decreasing the time and radius intervals. Experience has shown that the over-all thermal energy balance is quite sensitive to the use of improper or inadequate approximations and a close check on this quantity was maintained. For the models reported here variations within the thermal energy balance were less than 0.1%.

LIMITATIONS OF HOMOGENEOUS MODELS

It must be clearly recognized that these models serve only to demonstrate the effects of melting and cannot be considered to be realistic models of the Earth as they fail to satisfy the experimentally determined requirements of a solid mantle at the present time and the observed mean surface heat flow of approximately 64 erg/m^2sec [Lee and MacDonald, 1963]. The heat flow from the surface of the convective models after 4.5 billion years totals 25.4 erg/m^2sec for the chondritic and 17.0 erg/m^2sec for the terrestrial cases. These low values are a natural consequence of the fact that the radioactives were assumed to remain uniformly distributed. This is certainly not the case within the Earth.

The known concentration of radioactive materials within the crust demands a highly efficient differentiation process if the Earth is assumed to have had a uniform initial composition. It is difficult to visualize any sort of efficient differentiation process as occurring through solid material of such high densities and thus it would seem that at least partial melting should be a requirement for operation of the differentiation process. As Figure 6 shows for a chondritic Fig. 6

model and Figure 7 for a terrestrial model, these melting models can indicate, for a given set of conditions, the times and depths at which melting and, hence, on the above assumption, differentiation begins. Thus, calculations for a differentiated planet are possible. Preliminary calculations have been carried out for a differentiating Moon and models of the Earth and other planets are currently under construction. Fig. 7

SUMMARY

Investigation into the problem of melting within the Earth revealed a strong possibility that the melting temperatures within the outer layers of the Earth have been exceeded at some time in the Earth's history. Fluid convection should then have occurred and been an efficient process of heat transfer. We have developed numerical techniques for calculating the temperature distribution within a spherical body which melts according to a simple melting curve and which convects with the limiting case of high efficiency. These techniques were applied to the outer layers of the Earth and the results and limitations discussed. The importance of these techniques as a tool for studying the problem of differentiation of radioactive materials within the Earth has been indicated.

APPENDIX

DISCUSSION OF NUMERICAL TECHNIQUES

Given the equation describing the temperature distribution in a spherically symmetric body

$$\rho C_p \frac{\partial T}{\partial t} = K\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) + \frac{\partial K}{\partial r}\frac{\partial T}{\partial r} + A$$

standard techniques are used to replace this equation by a difference equation [see <u>Herriot</u>, 1963].

Taking a rectangular lattice in the rt plane in which the lattice points are at $r_m = r_o + m\,\Delta r$ and $t_n = t_o + n\,\Delta t$, where m and n are integers, and using the relations

$$\frac{\partial T}{\partial t} = \frac{T_m^{n+1} - T_m^n}{\Delta t} \qquad \frac{\partial T}{\partial r} = \frac{T_{m+1}^n - T_{m-1}^n}{2\,\Delta r}$$

$$\frac{\partial K}{\partial r} = \frac{K_{m+1}^n - K_{m-1}^n}{2\,\Delta r} \qquad \frac{\partial^2 T}{\partial r^2} = \frac{T_{m+1}^n - 2T_m^n + T_{m-1}^n}{(\Delta r)^2}$$

we obtain the governing difference equation,

$$T_m^{n+1} = T_m^n + \frac{\Delta t}{\rho C_p}\left\{\frac{1}{m(\Delta r)^2}\left[\frac{m}{4}\left(K_{m+1}^n - K_{m-1}^n\right) + (m+1)K_m^n\right] T_{m+1}^n\right.$$

$$\left. - 2mK_m^n T_m^n - \left[\frac{m}{4}\left(K_{m+1}^n - K_{m-1}^n\right) - (m-1)K_m^n\right] T_{m-1}^n + A_m^n\right\}$$

The boundary conditions to be satisfied are

$$T_m^0 = f(m\,\Delta r)$$

$$T_M^n = g(n\,\Delta t) \qquad r = M \text{ (at surface)}$$

$$T_{r_0}^n = \frac{5}{2}\,T_{r_0+\Delta r}^n - 2T_{r_0+2\,\Delta r} + \frac{1}{2}\,T_{r_0+3\,\Delta r} \qquad r = r_0 \text{ (at inner surface)}$$

In order to maintain a stable solution, the time interval was fixed at

$$\Delta t = \frac{\rho_s C_p (\Delta r)^2}{2K_m^n}$$

For a discussion of the convergence of heat flow solutions see Kunz [1957]. Figure 8 illustrates the method devised for accounting for the heat of fusion during the melting process. Three states of material are defined as Fig. 8

Phase I;	$H_{m_{res}}^n = H_{in}$	nonmolten
Phase II;	$0 < H_{m_{res}}^n = H_m^{n-1} - C_s(T_m^n - T_m^n) < H_{in}$	partially molten
Phase III;	$H_{m_{res}}^n = 0$	completely molten

where $H_{m_{res}}^n$ is the residual heat of fusion and H_{in} is the total heat of fusion.

The modifications to figure 8 to include the effects of simulated convection are indicated by the numbers 1 through 4 at the appropriate points where the following changes are to be inserted:

1. Add $T_{m-1_{con}}^n$ to T_m^n, where $T_{m-1_{con}}^n$ is calculated according to 2, 3, or 4 for the previous lattice point

2. $$T^n_{m_{con}} = \frac{(T^n_m - T^*_m)(r^2_{m-1}\rho_{m-1})}{r^2_m \rho_m}$$

3(a). $$T^n_{m_{con}} = \frac{(T^n_{m_2} - T^*_m)(r^2_{m-1}\rho_{m-1})}{r^2_m \rho_m}$$, where $T^n_{m_2}$ is computed as indicated in Figure 8

3(b). Set $T^n_m = T^*_m$

4. $$T^n_{m_{con}} = 0$$

REFERENCES

Alexeyeva, K. N., Physical properties of stoney meteorites and their interpretation in the light of hypothesis of meteoritic origin, Meteoritica, 16, 67-77, 1958.

Allan, D. W., and J. A. Jacobs, The melting of asteroids and the origin of meteorites, Geochim. et Cosmochim. Acta, 9, 256-272, 1956.

Benfield, A. E., The temperatures in an accreting earth, Trans. Am. Geophys. Union, 31(1), 53-5[illegible], 1950.

Birch, Francis, Density and composition of mantle and core, J. Geophys. Res., 69(20), 4377-4388, 1964.

Birch, Francis, Speculations on the earth's thermal history, The Geol. Soc. Am. Bull., 76(2), 133-154, 1965.

Birch, Francis, J. F. Schairer, and H. C. Spicer, Editors, Handbook of Physical Constants, Geol. Soc. Am., Special Paper 36, 1942.

Boyd, F. R., and J. L. England, Effect of pressure on the melting of diopside, $CaMgSi_2O_6$ and albite, $NaAlSi_3O_8$, in the range up to 50 kilobars, J. Geophys. Res., 68(1), 311-323, 1963.

Boyd, F. R., J. L. England, and Brian T. C. Davis, Effects of pressure on the melting and polymorphism of enstatite, $MgSiO_3$, J. Geophys. Res., 69(10), 2101-2109, 1964.

Clark, Sydney P., Jr., Variation of density in the earth and the melting curve in the mantle, in The Earth Sciences, edited by Thomas W. Donnelly, pp. 5-42, The University of Chicago Press, Chicago, 1963.

Davis, B. T. C., and J. L. England, The melting of forsterite up to 50 kilobars, J. Geophys. Res., 69(6), 1113-1116, 1964.

Elsasser, W. M., Early history of the earth, in Earth Science and Meteoritics, edited by J. Geiss, pp. 1-30, Interscience Pub., New York, 1963.

Fish, Robert A., Gordon C. Goles, and Edward Anders, The record in the meteorites. III. On the development of meteorites in asteroidal bodies, Astrophys. J., 132(1), 243-258, 1960.

Herriot, John George, Methods of Mathematical Analysis and Computation, John Wiley and Sons, Inc., New York, 1963.

Hess, H. H., Stillwater igneous complex, Montana Geol. Soc. Am. Memoir 80, Waverly Press Inc., Baltimore, p. 145, 1960.

Jeffreys, H., The Earth: Its origin, history, and physical constitution, 4th edition, Cambridge University Press, Cambridge, 1962.

Kaula, W. M., Tidal dissipation by solid friction and the resulting orbital evolution, Rev. Geophys., 2, 661-685, 1964.

Knopoff, L., The convection current hypothesis, Rev. Geophys., 2, 89-123, 1964.

Kunz, K. S., Numerical Analysis, McGraw-Hill Book Co., 1957.

Lee, W. H. K., and Gordon J. F. MacDonald, The global variation of terrestrial heat flow, J. Geophys. Res., 68(4), 6481-6492, 1963.

Levin, B. J., Thermal history of the moon, in The Moon, edited by Z. Kopal, and Z. K. Mikhailov, pp. 157-167, Academic Press, London, 1962.

Lubimova, H. A., Heating of the earth's interior during the earth's formation, Izvestiya Akademii Nauk SSSR, ser geofiz, no. 5, 416-424, 1955.

Lubimova, H. A., Thermal history of the earth with consideration of the variable thermal conductivity of its mantle, Geophys. J. Royal Astronomical Soc., 1, 115-134, 1958.

MacDonald, Gordon J. F., Calculations on the thermal history of the earth, J. Geophys. Res., 64(11), 1967-2000, 1959.

MacDonald, Gordon J. F., The deep structure of continents, Rev. Geophys., 1(4), 587-665, 1963.

MacDonald, Gordon J. F., Dependence of the surface heat flow on the radioactivity of the earth, J. Geophys. Res., 69(14), 2933-2946, 1964.

Safronov, V. S., The growth of terrestrial planets, Problems of Cosomogony, 6, pp. 71-89, Moscow Acad. Sci., USSR, 1958.

Strong, H. M., High temperature methods at high pressure, in Modern Very High Pressure Techniques, edited by R. H. Wentorf, Jr., pp. 93-117, Robert Henry Butterworths, Jr., Inc., Washington, 1962.

Uffen, Robert J., A method of estimating the melting-point gradient in the earth's mantle, Trans. Am. Geophys. Union, 33(6), 893-896, 1952.

Urey, Harold Clayton, The Planets, Their Origin and Development, Yale University Press, New Haven, 1952.

Urey, Harold C., and Bertram Donn, Chemical heating for meteorites, Astrophys. J., 124(1), 307-310, 1956.

Vening Meinesz, Felix Andries. The Earth's Crust and Mantle, Elsevier Pub. Co., Amsterdam, 1964.

Wasserburg, G. J., Gordon J. F. MacDonald, F. Hoyle, and William A. Fowler, Relative contributions of uranium, thorium, and potassium to heat produced in the earth, Science, 143, 465-467, 1964.

Yoder, H. S., Jr., and C. E. Tilley, Origin of basalt magmas, an experimental study of natural and synthetic rock systems, J. Petrology, 3(3), 342-532, 1962.

TABLE I. Abundances of Radioactive Heat Sources (in 10^{-8} g/g)

Radioactive Nuclide	Chondritic Model (MacDonald, 1959)	Terrestrial Model (Wasserburg et al., 1964)
U^{238}	1.092	2.239
U^{235}	0.0079	0.0162
Th^{232}	4.4	8.344
K^{40}	9.52	2.68

TABLE 2. Parameters For Thermal Models

Heat capacity, C_p		1.3 joules/g-deg
Lattice conductivity, c		0.025 joule/cm-sec-deg
Heat of fusion, H_{in}		400 joules/g
Index of refraction, n		1.7
Opacity, ϵ		100 cm^{-1}
Surface temperature, T^0		$0^\circ C$
Radioactive heat generation, B_j	U^{238}	2.97 joules/g-yr
	U^{235}	18.0 joules/g-yr
	Th^{232}	0.82 joule/g-yr
	K^{40}	0.94 joule/g-yr
Decay constant, λ_j	U^{238}	1.54×10^{-10} yr^{-1}
	U^{235}	9.71×10^{-10} yr^{-1}
	Th^{232}	0.499×10^{-10} yr^{-1}
	K^{40}	5.5×10^{-10} yr^{-1}

TABLE 3. Temperature Distribution After 4.5 Billion Years Using a Modified Clark Melting Curve

	Uniform Chondritic Model			Uniform Terrestrial Models		
Depth (km)	Temp. (°C) nonmelting	Temp. (°C) melting	Temp. (°C) melting plus simulated convection	Temp. (°C) nonmelting	Temp. (°C) melting	Temp. (°C) melting plus simulated convection
0	0	0	0	0	0	0
100	773	720	657	602	562	657
200	1423	1329	1242	1118	1043	1237
300	1920	1793	1520	1531	1430	1520
400	2286	2136	1620	1847	1729	1620
500	2556	2393	1780	2085	1955	1780
600	2755	2586	1980	2261	2122	1980
700	2901	2730	2235	2389	2235	2198
800	3008	2839	2523	2480	2378	2355
900	3085	2929	2741	2544	2477	2465
1000	3140	3011	2902	2587	2545	2539
1100	3179	3083	3019	2616	2591	2588
1200	3205	3139	3103	2635	2620	2619
1300	3224	3182	3163	2624	2639	2639
1400	3237	3216	3208	2654	2651	2651
1500	3247	3246	3245	2659	2659	2659

FIGURE LEGENDS

Figure 1.- Theoretical melting curves for the outer layers of the Earth.

Figure 2.- Temperature distribution after 4.5 billion years for uniform chondritic models.

Figure 3.- Temperature distribution after 4.5 billion years for uniform terrestrial models.

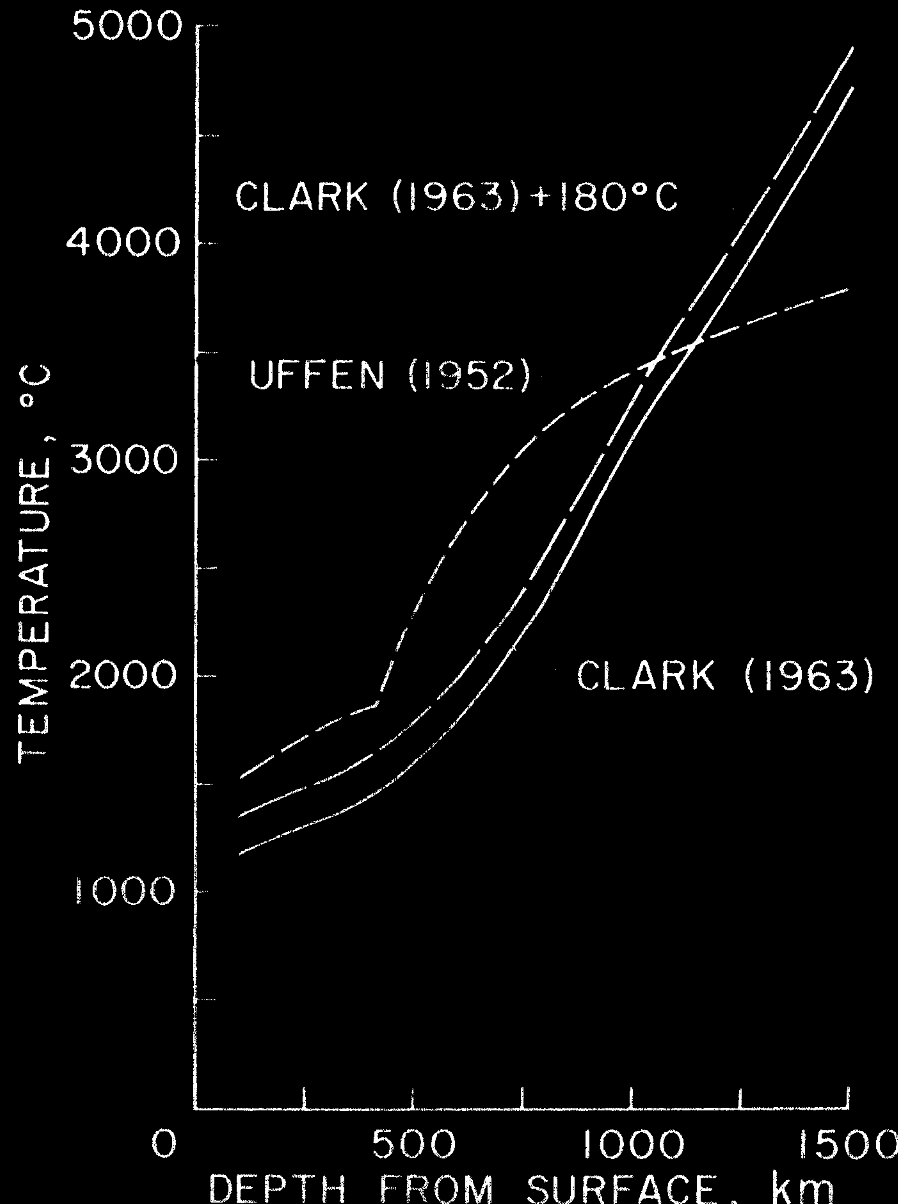
Figure 4.- Temperature distribution after 4.5 billion years for uniform chondritic models.

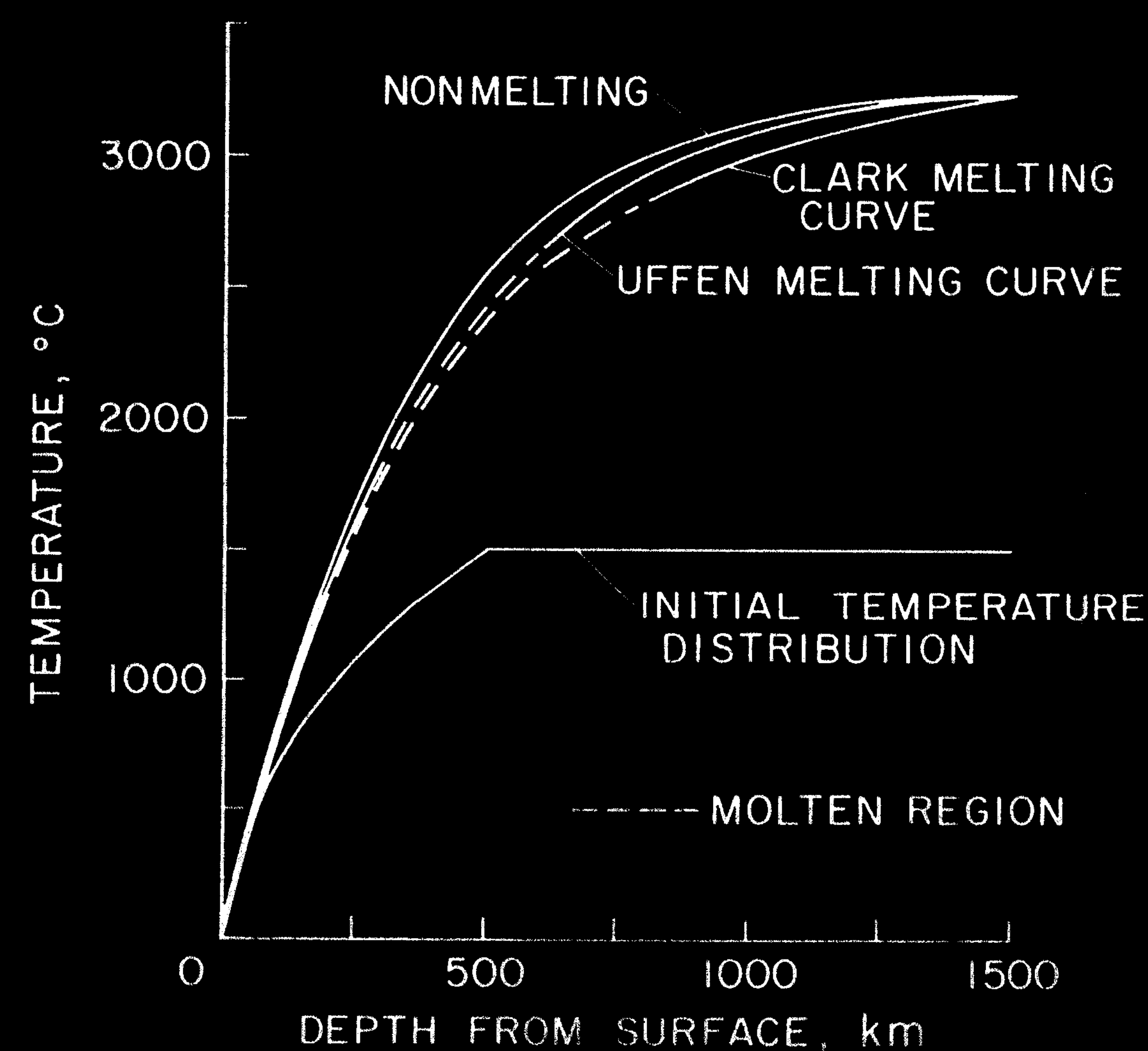
Figure 5.- Temperature distribution after 4.5 billion years for uniform terrestrial models.

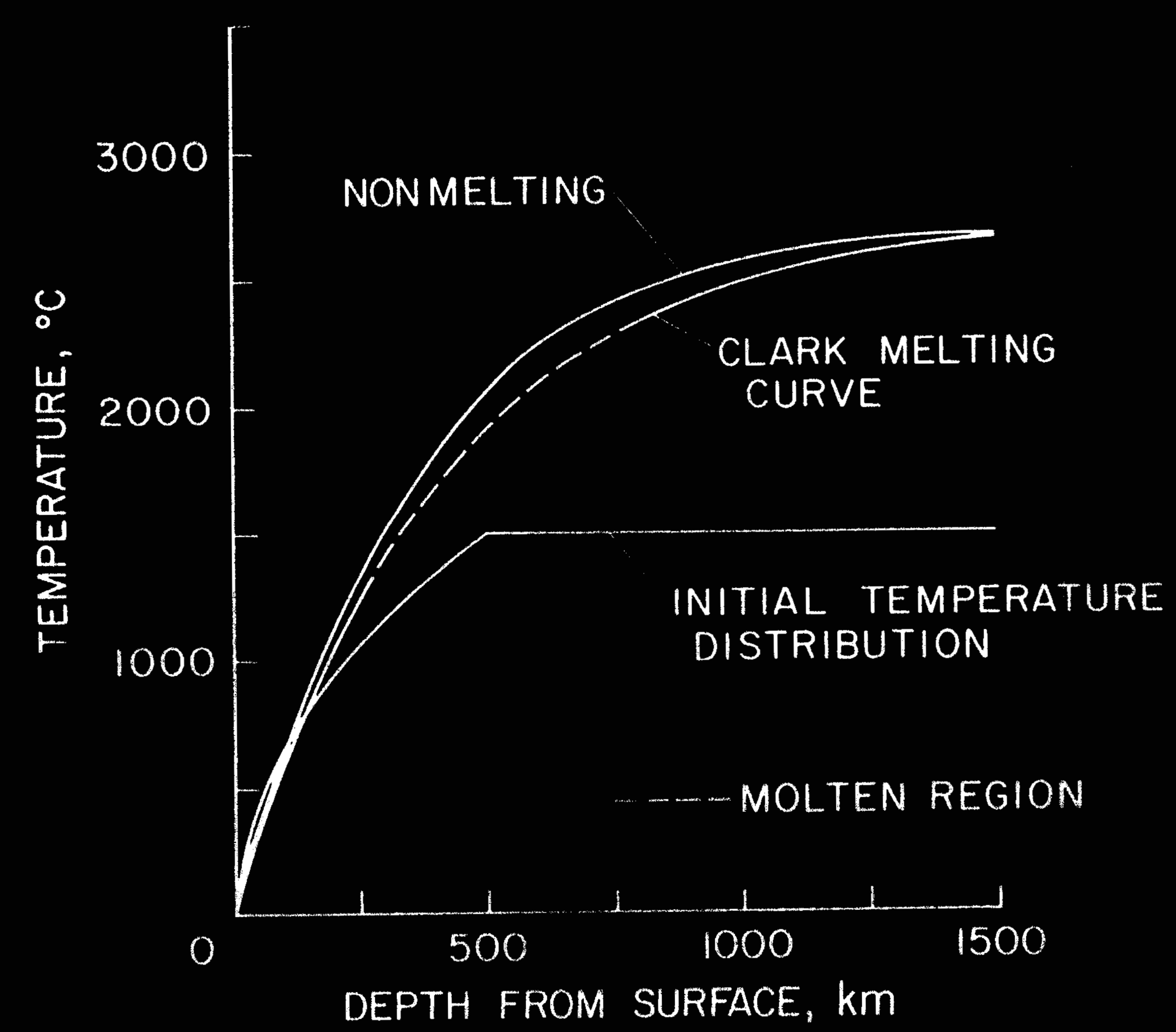
Figure 6.- Variation of temperature with time for a uniform chondritic model using the modified Clark melting curve.

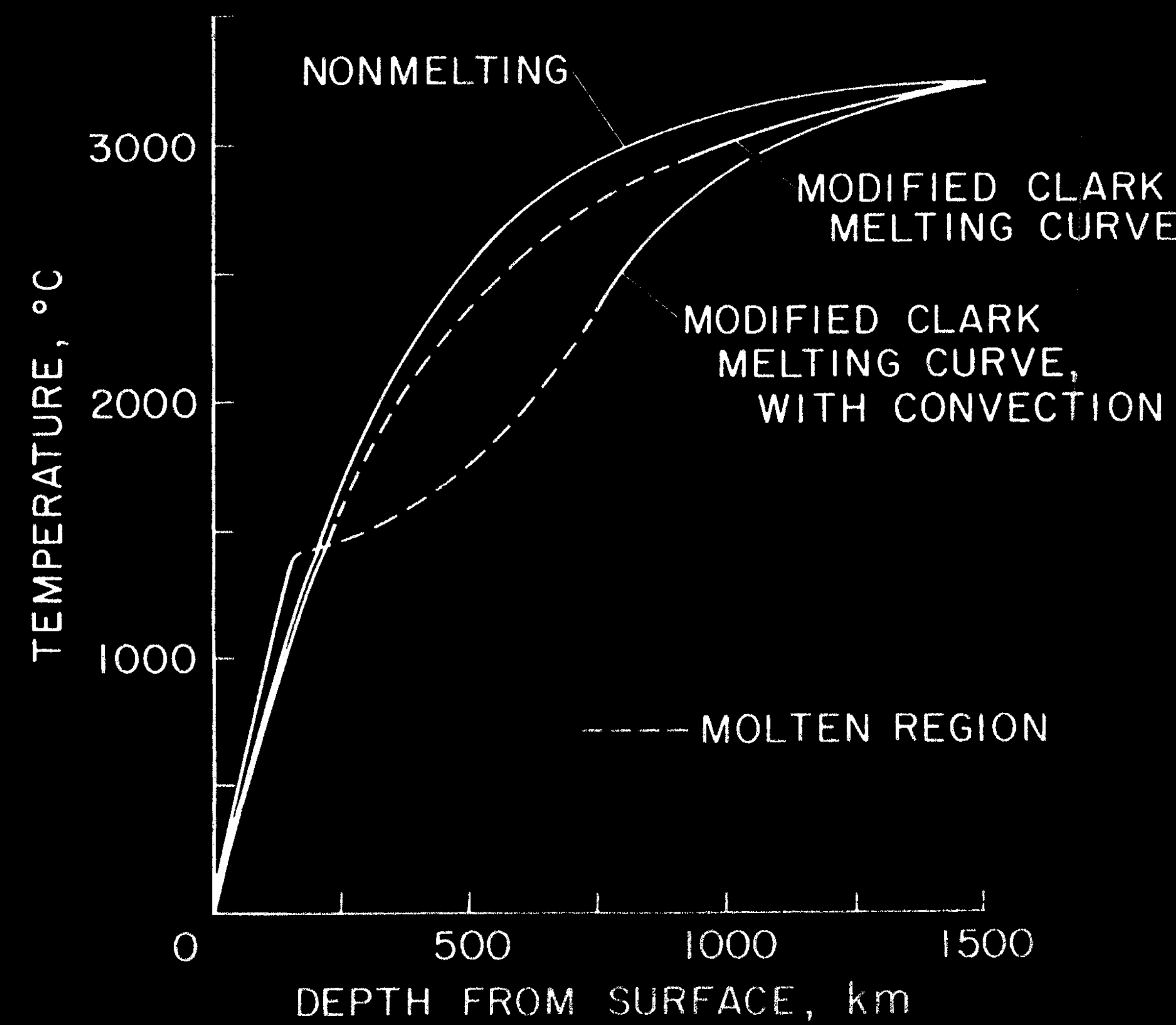
Figure 7.- Variation of temperature with time for a uniform terrestrial model using the modified Clark melting curve.

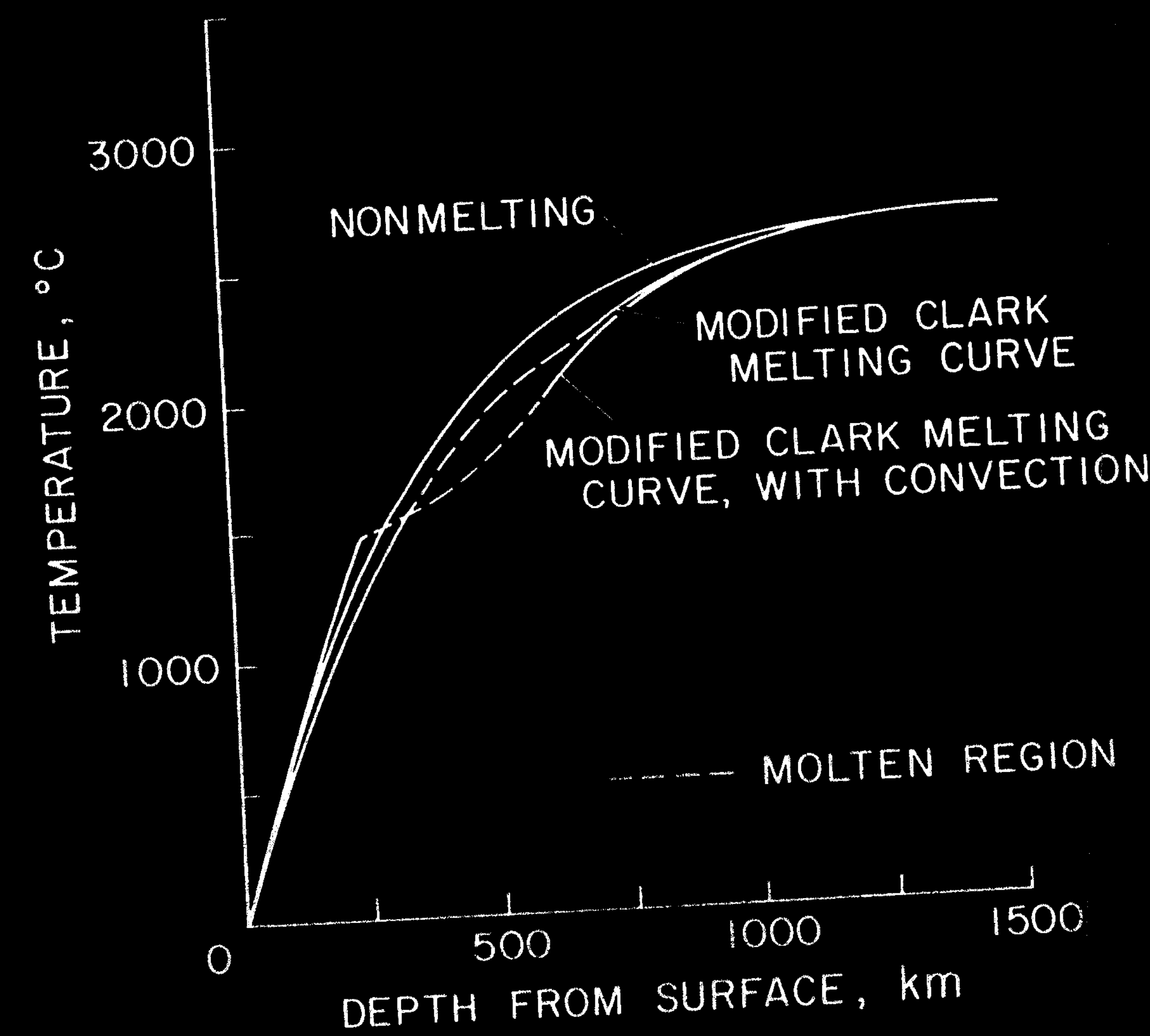
Figure 8.- Numerical technique for calculation of melting models.

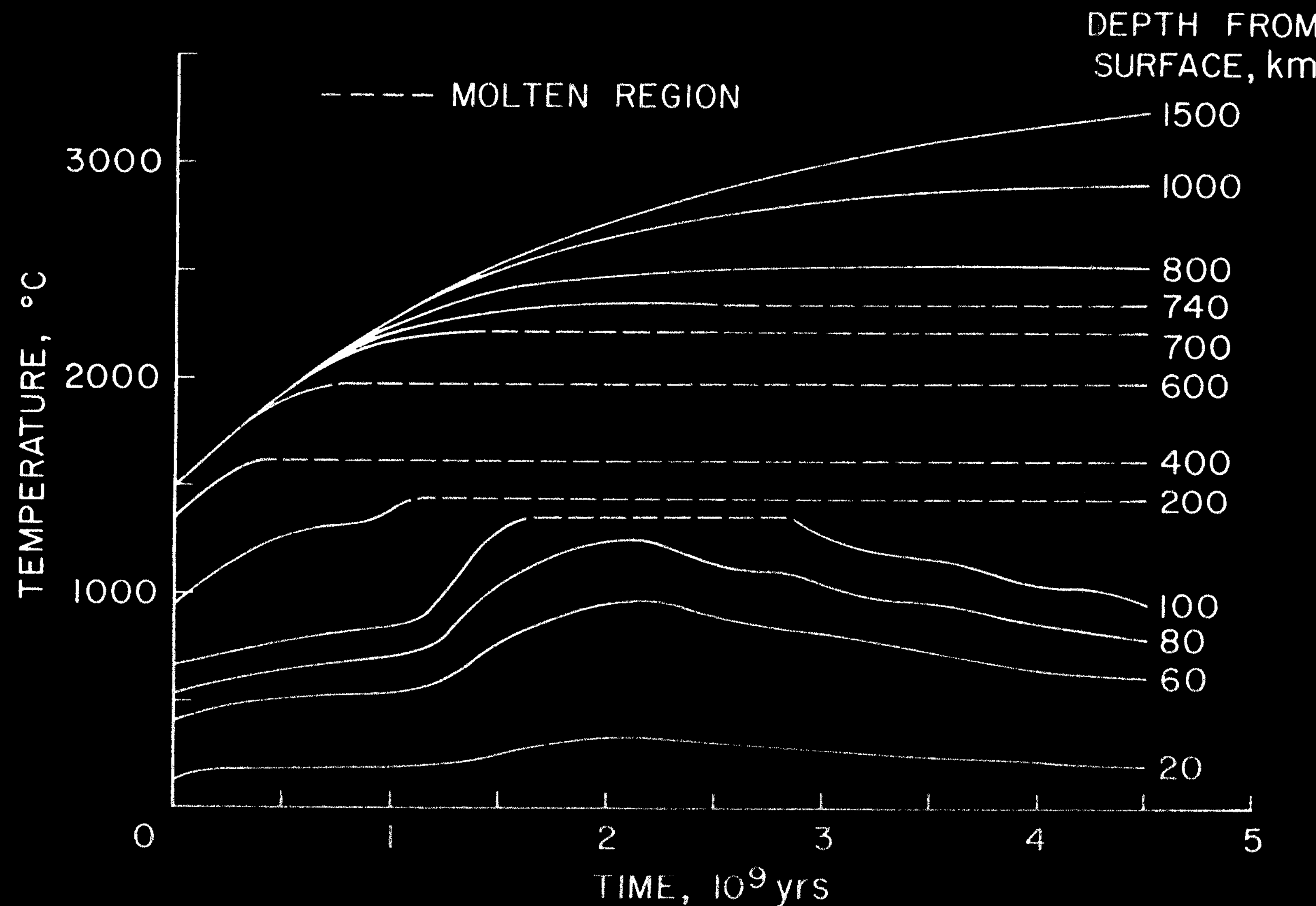

TEMPERATURE, °C
5000
4000
3000
2000
1000
0
500
1000
1500
DEPTH FROM SURFACE, km
CLARK (1963)+180°C
UFFEN (1952)
CLARK (1963)


TEMPERATURE, °C
3000
2000
1000
0
500
1000
1500
DEPTH FROM SURFACE, km
NONMELTING
CLARK MELTING CURVE
UFFEN MELTING CURVE
INITIAL TEMPERATURE DISTRIBUTION
MOLTEN REGION

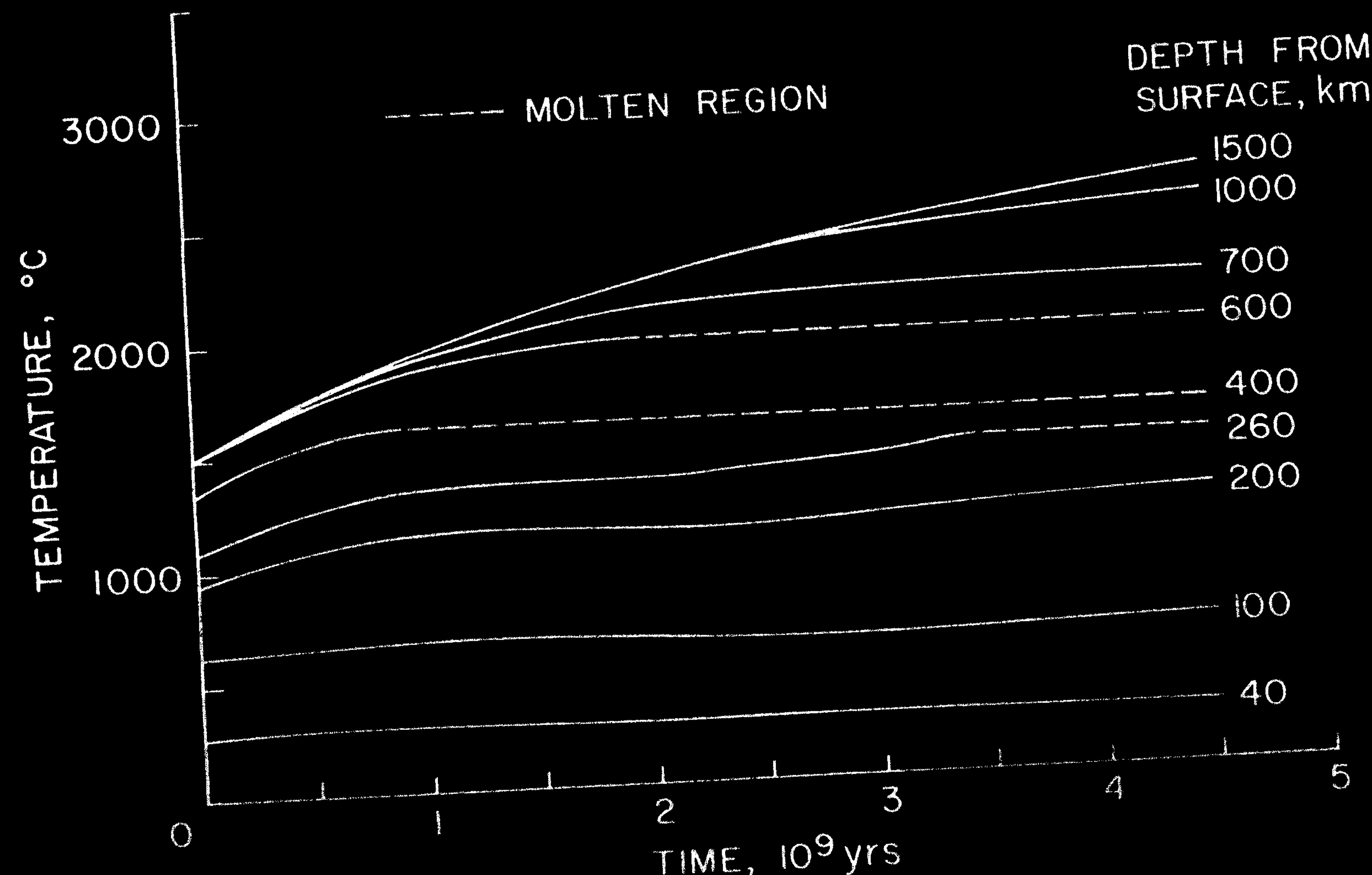

TEMPERATURE, °C
3000
2000
1000
0
500
1000
1500
DEPTH FROM SURFACE, km
NONMELTING
CLARK MELTING
CURVE
INITIAL TEMPERATURE
DISTRIBUTION
MOLTEN REGION


NONMELTING
MODIFIED CLARK MELTING CURVE
MODIFIED CLARK MELTING CURVE, WITH CONVECTION
MOLTEN REGION
TEMPERATURE, °C
3000
2000
1000
0
500
1000
1500
DEPTH FROM SURFACE, km

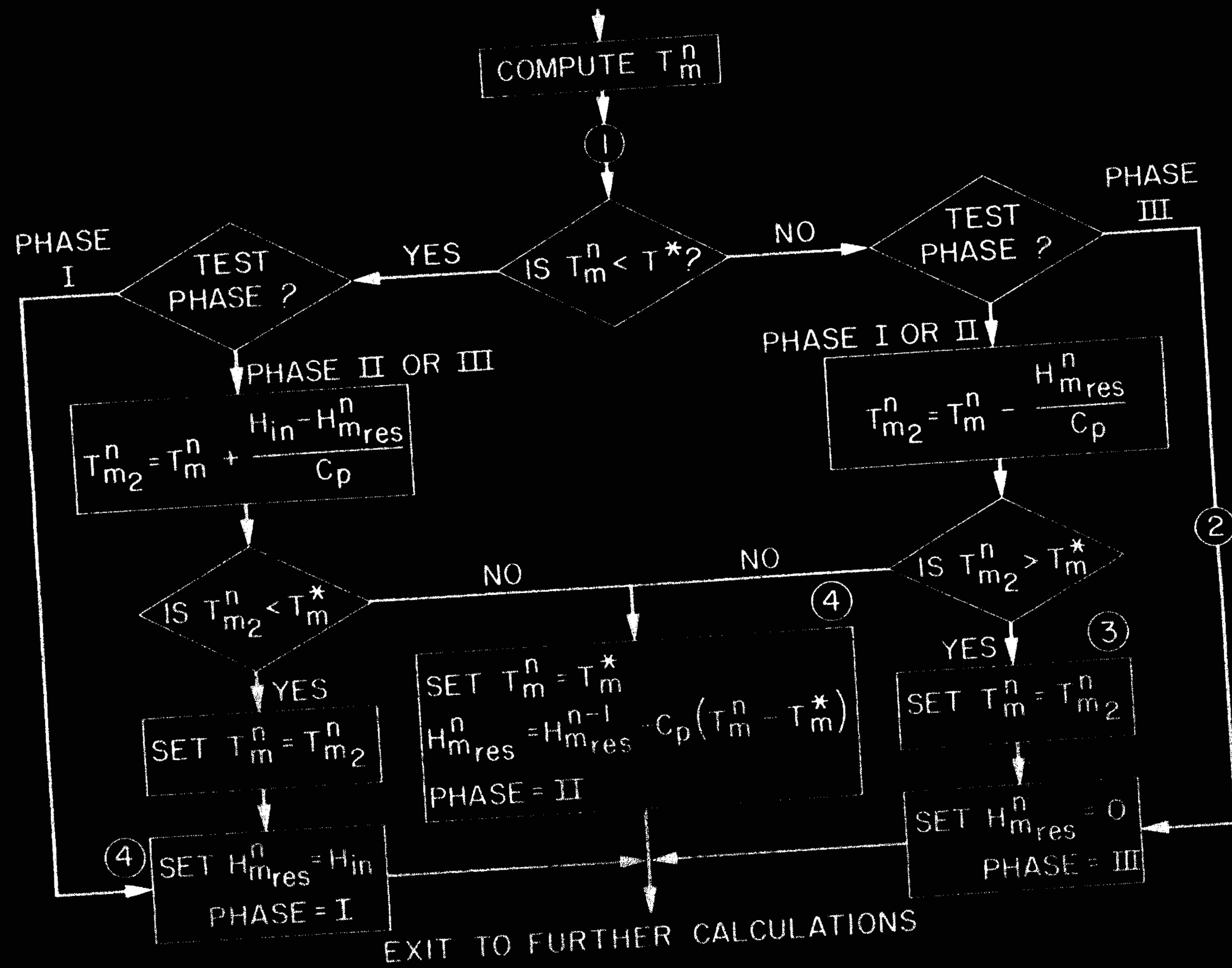

TEMPERATURE, °C
3000
2000
1000
0
NONMELTING
MODIFIED CLARK
MELTING CURVE
MODIFIED CLARK MELTING
CURVE, WITH CONVECTION
MOLTEN REGION
500
1000
1500
DEPTH FROM SURFACE, km



---- MOLTEN REGION
DEPTH FROM SURFACE, km
1500
1000
800
740
700
600
400
200
100
80
60
20
TEMPERATURE, °C
3000
2000
1000
0
1
2
3
4
5
TIME, 10^9 yrs



TEMPERATURE, °C
3000
2000
1000
0
1
2
3
4
5
TIME, 10^9 yrs
MOLTEN REGION
DEPTH FROM SURFACE, km
1500
1000
700
600
400
260
200
100
40



COMPUTE T_m^n
1
IS $T_m^n < T^*$?
YES
NO
PHASE I
TEST PHASE ?
PHASE II OR III
$T_{m_2}^n = T_m^n + \frac{H_{in} - H_{m_{res}}^n}{C_p}$
IS $T_{m_2}^n < T_m^*$
YES
SET $T_m^n = T_{m_2}^n$
4
SET $H_{m_{res}}^n = H_{in}$
PHASE = I
NO
SET $T_m^n = T_m^*$
$H_{m_{res}}^n = H_{m_{res}}^{n-1} - C_p(T_m^n - T_m^*)$
PHASE = II
4
TEST PHASE ?
PHASE III
PHASE I OR II
$T_{m_2}^n = T_m^n - \frac{H_{m_{res}}^n}{C_p}$
2
IS $T_{m_2}^n > T_m^*$
NO
YES
3
SET $T_m^n = T_{m_2}^n$
SET $H_{m_{res}}^n = 0$
PHASE = III
EXIT TO FURTHER CALCULATIONS